Efficient Trajectory Modeling for Rocket/Moving Target Spatial Relationships

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Nomenclature

AZ = azimuth, positive east from north
QE = quadrant elevation from launcher horizontal
t = time interval from launch
T = time interval between launch and apogee
T_L = ephemeris time of launch

Introduction

AN efficient method of accurately computing the spatial relationships between sounding rockets and moving targets was developed for use in determining best launch times and directions for several different vehicles launched during the solar eclipse of March 7, 1970. The method, even though easily mechanized, has accuracy advantages over other approaches requiring the same amount of time, and time advantages over methods yielding the same accuracy. Calculations were carried out on a digital computer, but the method is partly amendable to hand calculation.

The computational procedure and logic flow of the method are shown in Fig. 1. The foundation of the method is a novel combination of some approximation formulas developed by Schaechter,[1] Eqs. (1–3), with a simple model of the trajectory, a parabola. The parabola is made an accurate representation of the trajectory, however, because it is fitted only to the post burnout portion of a flight and uses accurate apogee data in its determination. The method is a departure from the conventional use of parabolas for trajectory modeling in that all effects of a spherical, rotating Earth with variable gravitation and coriolis are included.

Algorithm

To apply the method in a particular case, two complete trajectory simulations are run at two representative QEs for the payload-rocket combination of interest. These simulations should be made with at least a three-dimensional, three-degree-of-freedom, spherical rotating Earth program in order to retain the inherent accuracy of the method. The apogee values found from the simulations (altitude, range, and time) are used to evaluate the constants of Eqs. (1–3). For example, the exponent and coefficient of the apogee altitude formula are evaluated as $N_H = \ln(H_1/H_2)/\ln(\sin QE_1/\sin QE_2)$ and $K_H = H_1/(\sin QE_1)^{N_H}$, where the numerical subscripts merely serve to identify the two trajectory simulations.

Equations (1–3) may then be used to calculate apogee height, range, and time for any desired QE:

$$H = K_H(\sin QE)^{N_H} \tag{1}$$

$$R = K_R \cos QE(\sin QE)^{N_R-} \tag{2}$$

$$T = K_T(\sin QE)^{N_T} \tag{3}$$

Trajectories corresponding to other QEs are then modeled

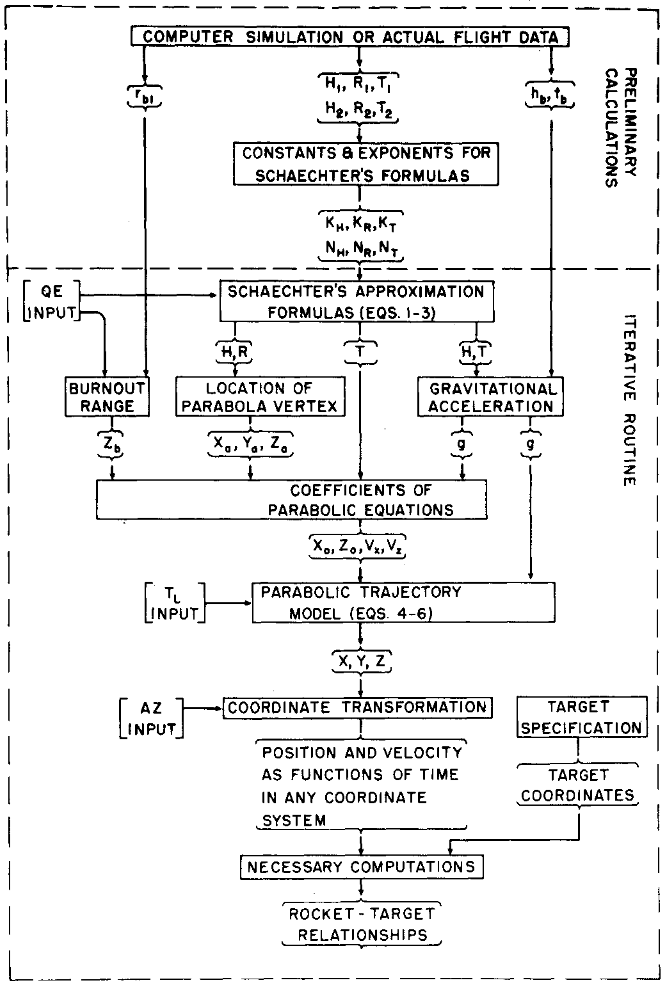

Fig. 1 Logic flow.

as parabolas using the familiar equations[2]

$$x = x_0 + v_x t - 0.5gt^2 \tag{4}$$

$$y = 0 \tag{5}$$

$$z = z_0 + v_z t \tag{6}$$

where the XYZ right-handed coordinate system is as shown in Fig. 2.

For any given set of launch parameters the coefficients of Eqs. (4–6) are determined from Eqs. (1–3) and from the time and position of final stage burnout. First the coordinates of the apogee point are calculated from the geometry of the situation as $x_a = H + r_e[(1 - \cos(R/r_e)]$, $y_a = 0$, and $z_a = r_e \sin(R/r_e)$. Then an average value of gravitational acceleration, g, is determined based upon the time interval and altitude interval between burnout and apogee. Burnout

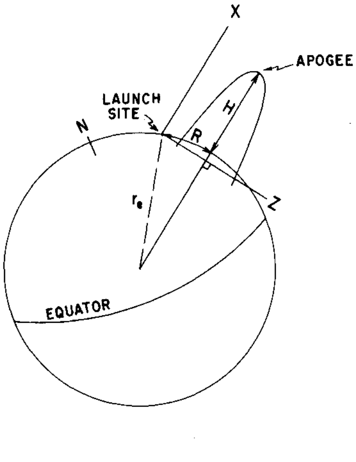

Fig. 2 Trajectory geometry.

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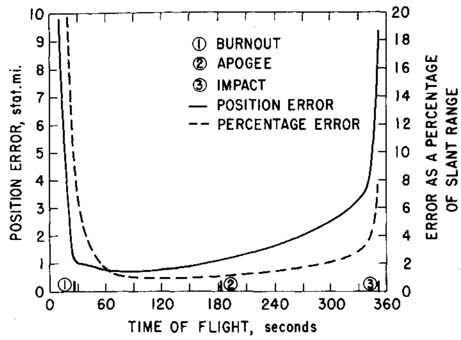

Fig. 3 Accuracy of the trajectory model.

altitude h_b and time t_b are assumed constant with respect to QE and are taken from the original trajectory simulations. Thus $g = 2(H - h_b)/(T - t_b)$.[2] Finally, burnout range, which cannot be assumed constant with respect to QE, is found from a simple linear approximation. If r_{b1} is the range given by the trajectory simulation for QE_1, the burnout range for any other QE is found from $r_b = r_{b1}\,(90° - QE)/(90° - QE_1)$. The coefficients of Eqs. (4–6) may then be evaluated as $y_0 = x_a - 0.5gT^2$, $v_x = gT$, $v_z = (z_a - r_b)/(T - t_b)$, and $z_0 = r_b - v_z t_b$.

Equations (4–6) allow explicit determination of payload position in space at any time, for any given launch QE, AZ, and time. Although of simple form, they accurately represent the trajectory because of their faithfulness to the actual physical situation. The equations yield a trajectory symmetric about the radius vector to apogee, a trajectory that goes through the predicted apogee point at the predicted time, and one that goes through the burnout point at the proper time.

Figure 3 indicates the accuracy of the trajectory model. Two complete trajectory simulations were run for QEs of 80° and 84°, at an azimuth of 140°, for the Nike Iroquois rocket from one particular launch site. The data from these two simulations were used to evaluate the constants of Eqs. (1–3). Then, using the trajectory model described here, position vs time was calculated for a QE of 76° at an AZ of 180°. A third complete trajectory simulation was then run for the same QE and AZ, and position data was compared. The position error plotted in Fig. 3 was calculated as the magnitude of the vector difference between the instantaneous position vectors as gained from the trajectory model and the trajectory simulation. Thus, the position error plotted is the total error and not merely a component of the error.

To calculate position in the trajectory model, azimuth of the trajectory plane XZ was assumed to be $AZ = 180° + \Delta AZ$; where ΔAZ was the change in azimuth of the rocket between launch and apogee as revealed by the trajectory simulation for QE_1. This correction is sufficient so long as a central value of azimuth is used in the trajectory simulation.

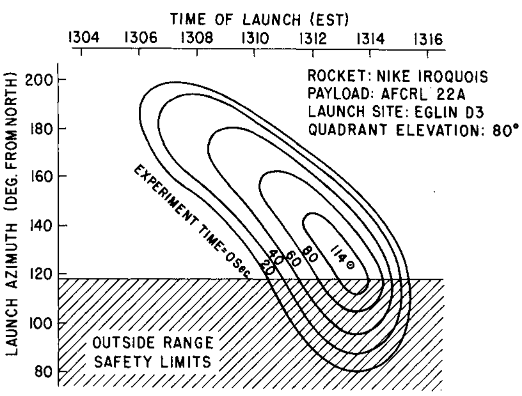

Fig. 4 Time spent in eclipse umbra (experiment time).

Application To Eclipse

Conventional Besselian Elements,[3] with numerical values taken from Ref. 4, were used to specify the eclipse conditions. Details of computations and coordinate systems are given elsewhere,[5] but Fig. 4 is presented as an example of the application of the method to one eclipse payload. Examination of a very large number of trajectories yielded the family of curves shown in less than a minute of computer execution time. In comparison, generating the same curves with repetitive trajectory simulations would have required hours.

References

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[2] Greenwood, D. T., *Principles Of Dynamics*, Prentice-Hall, N.J., 1965, pp. 65–70.

[3] *Explanatory Supplement To The Astronomical Ephemeris And The American Ephemeris And Nautical Almanac*, Nautical Almanac Offices of the U.K. and the U.S.A., London, 1961, pp. 211–222.

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Comparison of Hypersonic Aerodynamic Deceleration Systems Based on Gun-Tunnel Investigation

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Nomenclature

A_F, A_P	=	free and projected area of slotted flares, mm²
C_D, C_L	=	drag and lift coefficients
D	=	diameter of the cylinder, mm
H_s	=	spoiler height, mm
l, l_c	=	lengths of spike and body cylinder, mm
l_s	=	distance of the spoiler from the nose, mm
l_{sep}	=	length of the separated boundary layer, mm
M_∞	=	freestream Mach number
p_t	=	stagnation pressure, atm
$Re_D, Re_{x_{sep}}$	=	Reynolds number based on D and x_{sep}, respectively
T_t	=	stagnation temperature, °K
x_f, x_{sep}	=	distances between nose and junction of flare, and between nose and separation point, mm
β, θ_f	=	spike deflection and flare angles, deg
δ	=	measure of tangential extension of the spoiler, deg
$\vartheta_{sep}, \vartheta_{sh}$	=	separation angle and induced shock angle, deg

Introduction

THIS Note presents results of experimental investigations on aerodynamic deceleration devices for bodies composed of cylindrical main sections and various front sections, i.e.,

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